Topology-Aware Terrain Simplification

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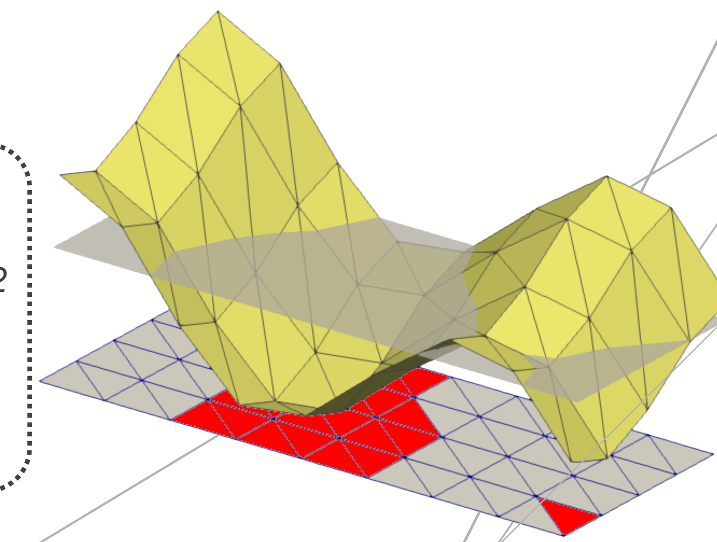
Preliminary Notions:

Terrain:

A triangulation T
- of a finite set of points $V \subseteq \mathbb{R}^2$
- endowed with a scalar field $f: V \longrightarrow \mathbb{R}$

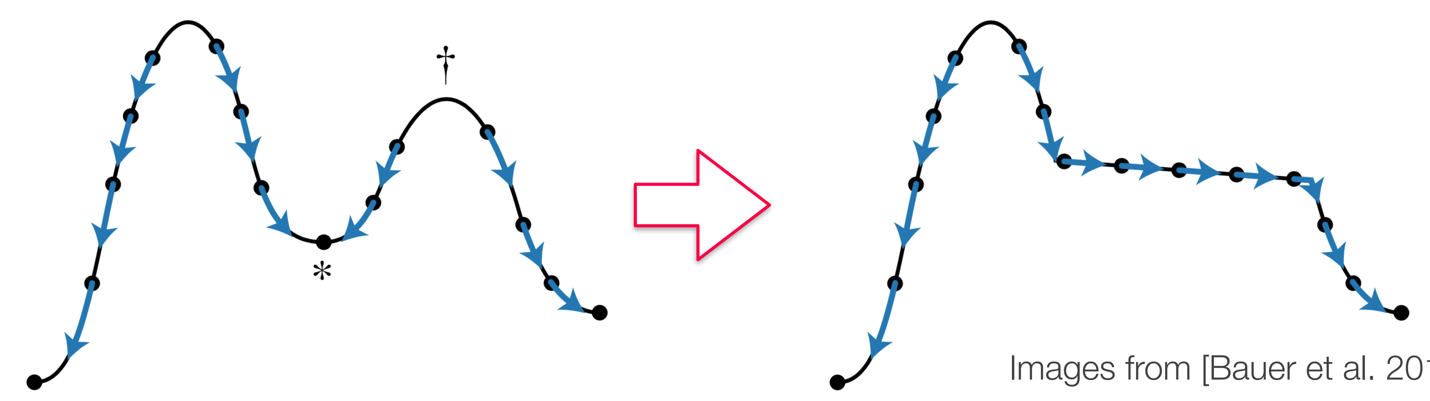


By filtering T through the height function f,
Persistent Homology enables the study of terrain morphology:

changes in homology ⟺ critical points of f

Topological Terrain Simplification:

Topological terrain simplification consists in the removal of



Images from [Bauer et al. 2012]

low persistence pairs of critical points considered as topological noise

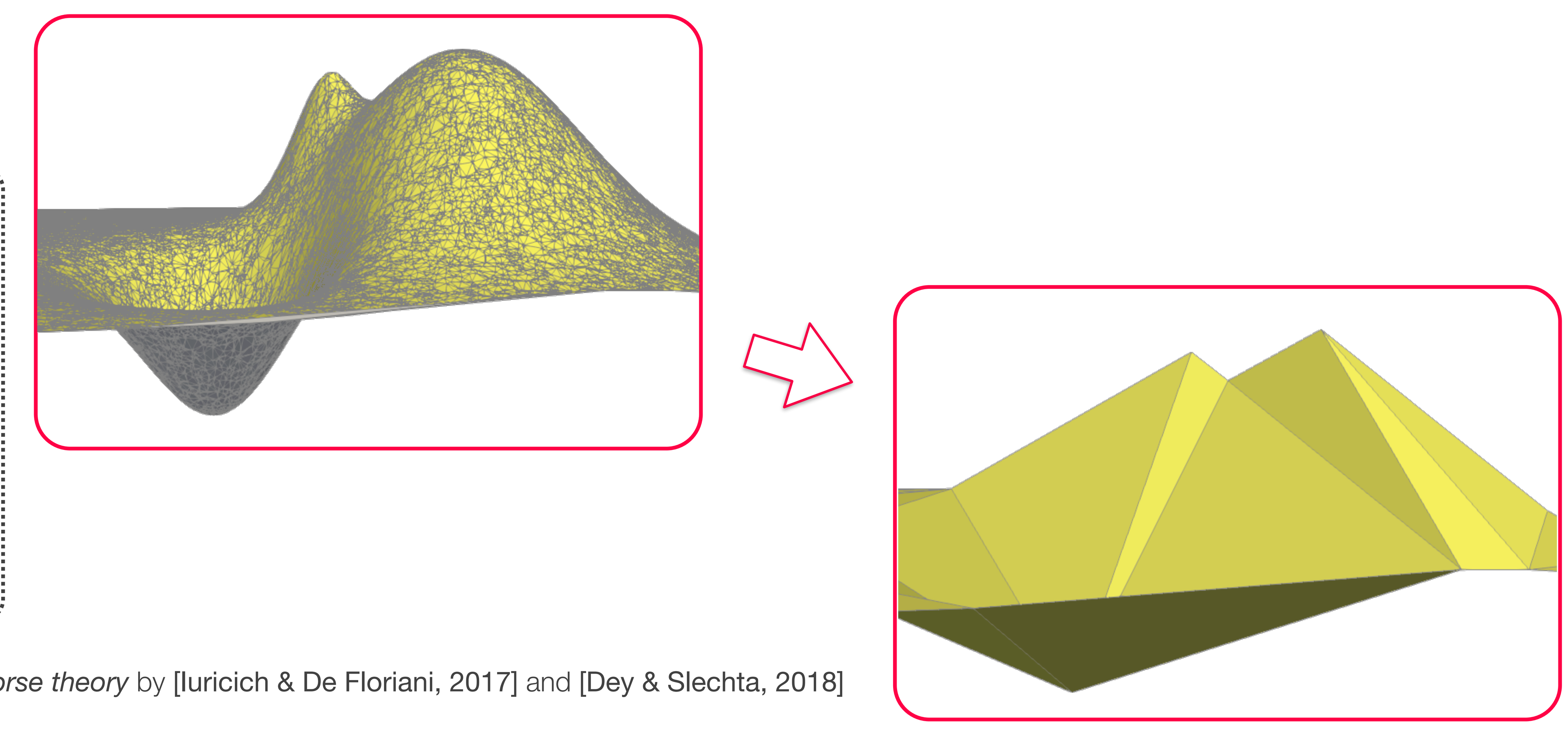
Geometrical Oversampling:

Regions free of critical points represented through a high number of triangles

Topology-Aware Terrain Simplification:

We propose a different approach:

- Reduce the number of triangles representing the terrain
- Maintain topological features and mesh quality

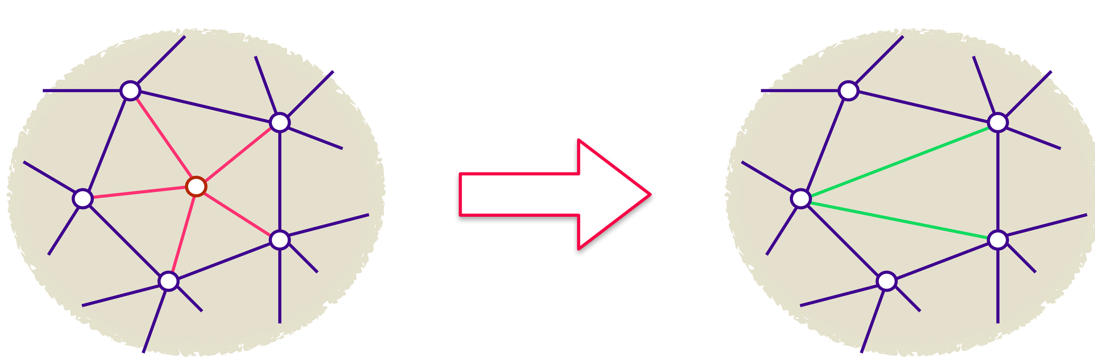


An analogous purpose has been considered in the framework of *discrete Morse theory* by [Iuricich & De Floriani, 2017] and [Dey & Slechta, 2018]

Algorithm Steps:

1. Perform vertex removals that:
   - preserve the persistent homology of the terrain
   - try to stay "as Delaunay as possible"

   as long as the reduced and the original terrains differ in L^∞-norm by at most a fixed threshold ϵ



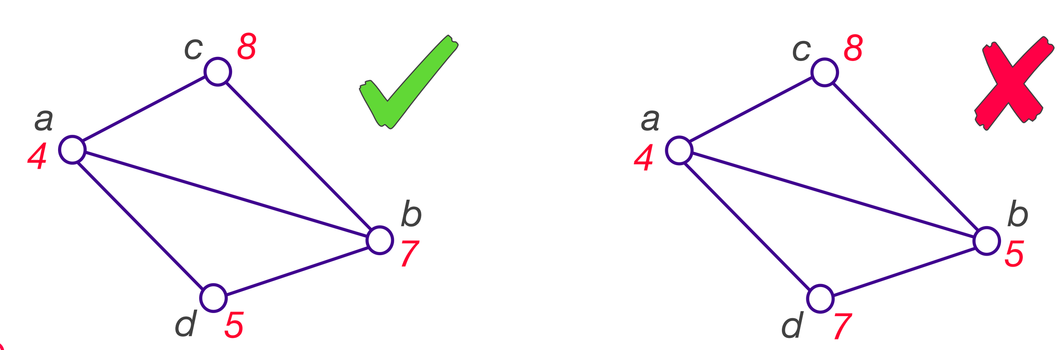
2. Compute constrained Delaunay triangulation improving mesh quality and preserving persistent homology

Edge Flip:

Given a quadrilateral *abcd*,

ab is feasible if $I_{ab} \cap I_{cd} \neq \emptyset$

where I_{uv} is the interval of $\mathbb{R}$ with extremal points $f(u)$ and $f(v)$

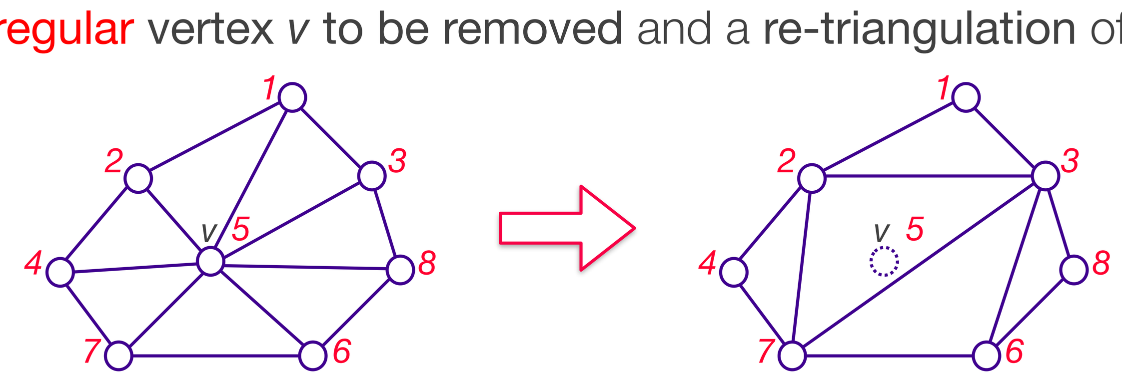


Lemma

Flipping a feasible edge does not change persistent homology

Vertex Removal:

Given a regular vertex v to be removed and a re-triangulation of its star



An edge ab of the re-triangulated star is persistence-aware if

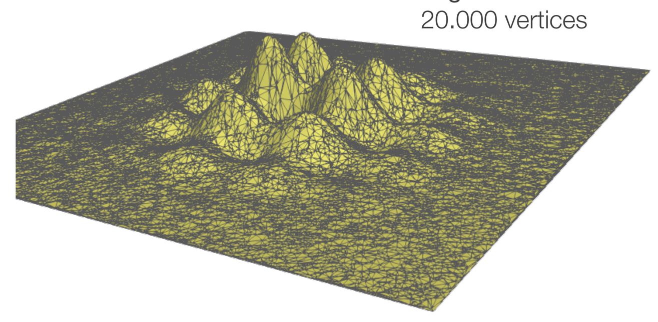
for all $c \in H_{ab}$, $I_{ab} \cap I_{cv} \neq \emptyset$

where H_{ab} consists of the vertices of the star of v belonging to the region of plane induced by ab and not containing v

Theorem

A re-triangulation of the star of v preserves persistent homology if all its edges are persistence-aware

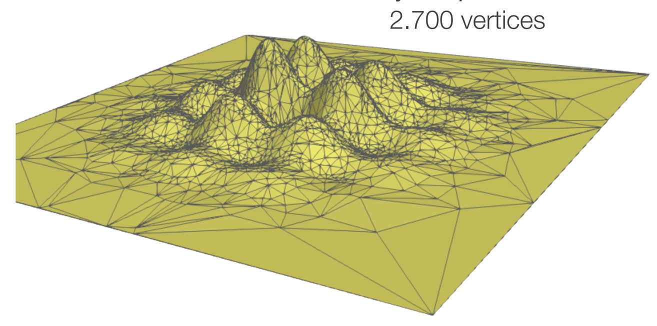
Experimental Results:



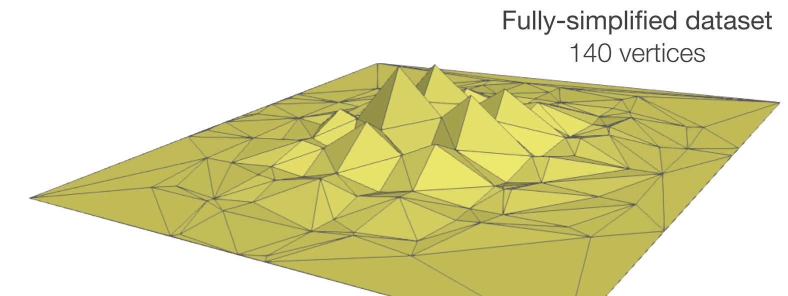
Original dataset
20.000 vertices

Dataset	# V	# V'	# C	Time
Gaussian	$2 \cdot 10^4$	140	134	30 s
Lake 1	10^5	12.000	9.400	4 min
Lake 2	10^5	1.300	240	4 min
Lake 3	$8 \cdot 10^5$	3.400	410	33 min

Datasets are courtesy of AIM@SHAPE repository



Partially-simplified dataset
2.700 vertices



Fully-simplified dataset
140 vertices

References:

- F. Iuricich, L. De Floriani. **Hierarchical Forman Triangulation: A multiscale model for scalar field analysis**. *Computers & Graphics* 66 (2017): 113-123.
- T. K. Dey, R. Slechta. **Edge contraction in persistence-generated discrete Morse vector fields**. *Computers & Graphics* (2018).
- U. Bauer, C. Lange, M. Wardetzky. **Optimal topological simplification of discrete functions on surfaces**. *Discrete & Computational Geometry* 47.2 (2012): 347-377.

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